



WORKSHEET

The Normal Distribution Question – Type 3

Question Description: In this question type they ask you to find or use an unknown mean and standard deviation. The key below shows you all the different styles they can do this.

Key:

- ◆ Finding the mean
- ◆ Finding the standard deviation
- ◆ Using equation relating μ and σ
- ◆ Finding both mean and standard deviation

QUESTIONS

1 s2015_p61

The lengths, in metres, of cars in a city are normally distributed with mean μ and standard deviation 0.714. The probability that a randomly chosen car has a length more than 3.2 metres and less than μ metres is 0.475. Find μ . [4]

2 s2015_p63

The weights, in grams, of onions in a supermarket have a normal distribution with mean μ and standard deviation 22. The probability that a randomly chosen onion weighs more than 195 grams is 0.128. Find the value of μ . [3]

3 w2016_p61

Packets of rice are filled by a machine and have weights which are normally distributed with mean 1.04 kg and standard deviation 0.017 kg.

The factory manager wants to produce more packets of rice. He changes the settings on the machine so that the standard deviation is the same but the mean is reduced to μ kg. With this mean the probability that a packet weighs less than 1 kg is 0.0388.

(iii) Find the value of μ . [3]

4 w2015_p62

The daily sales at another petrol station are X litres, where X is normally distributed with mean m and standard deviation 560. It is given that $P(X > 8000) = 0.122$.

(ii) Find the value of m . [3]

5 s2016_p61

The height of maize plants in Mpapwa is normally distributed with mean 1.62 m and standard deviation σ m. The probability that a randomly chosen plant has a height greater than 1.8 m is 0.15. Find the value of σ . [3]

6 w2015_p63

The time taken for cucumber seeds to germinate under certain conditions has a normal distribution with mean 125 hours and standard deviation σ hours.

(i) It is found that 13% of seeds take longer than 136 hours to germinate. Find the value of σ . [3]

7 s2015_p63

The heights of school desks have a normal distribution with mean 69 cm and standard deviation σ cm. It is known that 15.5% of these desks have a height greater than 70 cm.

(i) Find the value of σ . [3]

8 s2018_p63

The random variable X has the distribution $N(-3, \sigma^2)$. The probability that a randomly chosen value of X is positive is 0.25.

- (i) Find the value of σ . [3]

9 w2018_p63

The lifetimes, in hours, of a particular type of light bulb are normally distributed with mean 2000 hours and standard deviation σ hours. The probability that a randomly chosen light bulb of this type has a lifetime of more than 1800 hours is 0.96.

- (i) Find the value of σ . [3]

10 s2017_p63

- (b) The life of a particular type of torch battery is normally distributed with mean 120 hours and standard deviation s hours. It is known that 87.5% of these batteries last longer than 70 hours. Find the value of s . [3]

11 s2017_p61

- (a) The random variable X has a normal distribution with mean μ and standard deviation σ . You are given that $\sigma = 0.25\mu$ and $P(X < 6.8) = 0.75$.
(i) Find the value of μ . [4]

12 w2015_p62

- (b) The random variable Y is normally distributed with mean μ and standard deviation σ . Given that $\sigma = \frac{2}{3}\mu$, find the probability that a random value of Y is less than 2μ . [3]

13 s2018_p61

- (b) The random variable X has the distribution $N(\mu, \sigma^2)$, where $3\sigma = 4\mu$ and $\mu \neq 0$. Find $P(X < 3\mu)$. [3]

14 w2018_p62

- (b) The variable Y is normally distributed with mean μ and standard deviation σ , where $4\sigma = 3\mu$ and $\mu \neq 0$. Find the probability that a randomly chosen value of Y is positive. [3]

15 s2017_p63

- (a) The random variable X has the distribution $N(\mu, \sigma^2)$, where $\mu = 1.5\sigma$. A random value of X is chosen. Find the probability that this value of X is greater than 0. [3]

16 s2015_p62

- (b) The random variable X has the distribution $N(\mu, \sigma^2)$. It is given that $P(X < 7) = 0.2119$ and $P(X < 10) = 0.6700$. Find the values of μ and σ . [5]

17 w2015_p61

The random variable X has the distribution $N(\mu, \sigma^2)$. It is given that $P(X < 54.1) = 0.5$ and $P(X > 50.9) = 0.8665$. Find the values of μ and σ . [4]

18 w2017_p62

In Brigville the weights, in kilograms, of boys aged 16 years have a normal distribution. 99% of the boys weigh less than 97.2 kilograms and 33% of the boys weigh less than 55.2 kilograms.

- (iii) Find the mean and standard deviation of the weights of boys aged 16 years in Brigville. [5]

19 s2018_p61

- (a) The distance that car tyres of a certain make can travel before they need to be replaced has a normal distribution. A survey of a large number of these tyres found that the probability of this distance being more than 36 800 km is 0.0082 and the probability of this distance being more than 31 000 km is 0.6915. Find the mean and standard deviation of the distribution. [5]

20 w2018_p61

- (b) The lengths of fish of a particular species are modelled by a normal distribution. A scientist measures the lengths of 400 randomly chosen fish of this species. He finds that 42 fish are less than 12 cm long and 58 are more than 19 cm long. Find estimates for the mean and standard deviation of the lengths of fish of this species. [5]

21 w2019_p62

In another forest, the heights of another type of fir tree are modelled by a normal distribution. A scientist measures the heights of 500 randomly chosen trees of this type. He finds that 48 trees are less than 10 m high and 76 trees are more than 24 m high.

- (iii) Find the mean and standard deviation of the heights of trees of this type. [5]

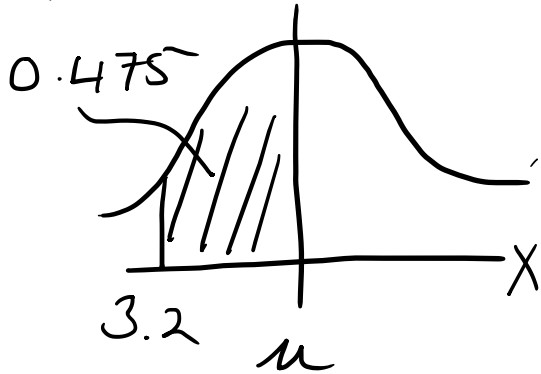
22 s2019_p62

It is known that 20% of male giant pandas in a certain area weigh more than 121 kg and 71.9% weigh more than 102 kg. Weights of male giant pandas in this area have a normal distribution. Find the mean and standard deviation of the weights of male giant pandas in this area. [5]

ANSWERS

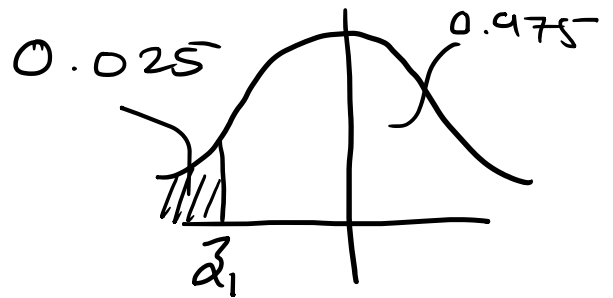
① $X \sim N(\mu, 0.714^2)$ $X = \text{length of cars}$

$$P(3.2 < X < \mu) = 0.475$$

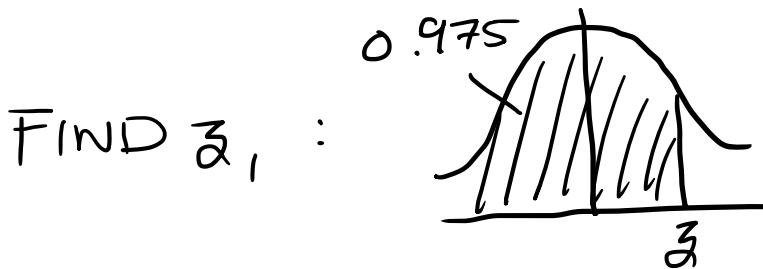


FROM THIS:

$$\begin{aligned} P(X < 3.2) &= 0.5 - 0.475 \\ &= 0.025 \end{aligned}$$



$$z_1 = \frac{3.2 - \mu}{0.714}$$



FIND z_1 :

$$\Phi(z) = 0.975$$

$$z = 1.96$$

$$\therefore z_1 = -1.96$$

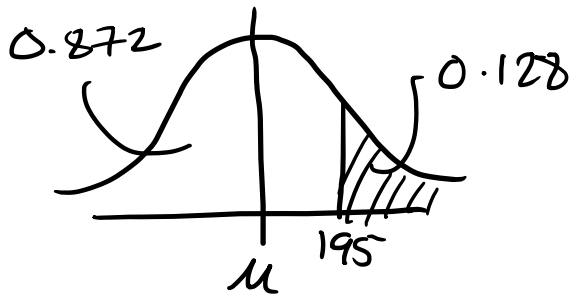
$$\therefore -1.96 = \frac{3.2 - \mu}{0.714}$$

$$\therefore \boxed{\mu = 4.605}$$

$$(2) \quad X \sim N(\mu, 22^2)$$

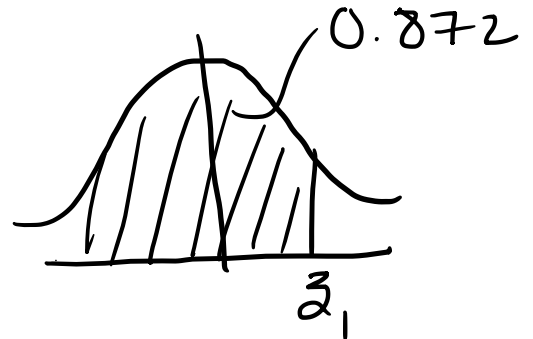
$X = \text{Weight of Unions}$

$$P(X > 195) = 0.128$$



$$\therefore Z_1 = \frac{195 - \mu}{22}$$

$\therefore$



$$\Phi(z_1) = 0.872$$

$$\therefore z_1 = 1.136$$

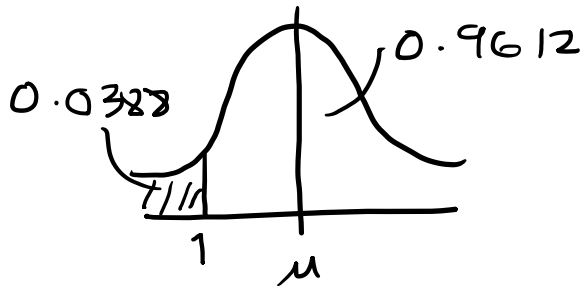
(FROM TABLE)

$$\therefore 1.136 = \frac{195 - \mu}{22}$$

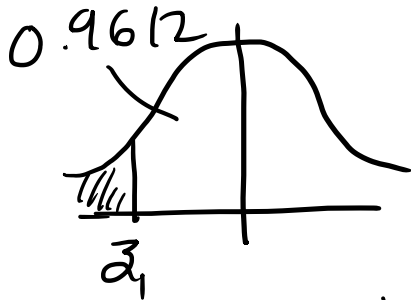
$$\therefore \boxed{\mu = 170g}$$

$$(3) \quad X \sim N(\mu, 0.017^2)$$

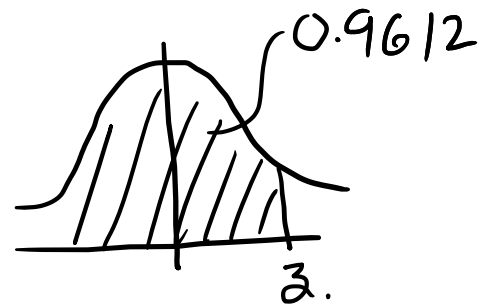
$$P(X < 1) = 0.0388$$



$$z_1 = \frac{1 - \mu}{0.017}$$



FIND z_1 :



$$\therefore \Phi(z) = 0.9612$$

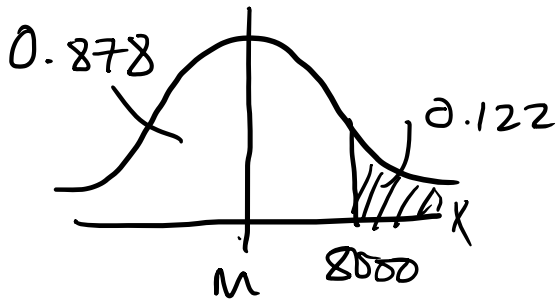
$$z = 1.765$$

$$\therefore z_1 = -1.765$$

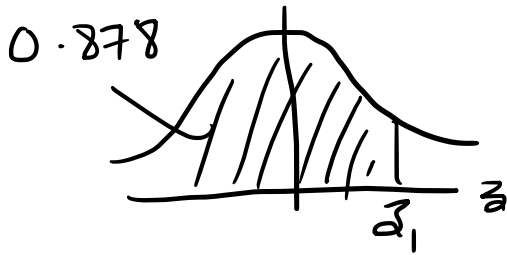
$$\therefore -1.765 = \frac{1 - \mu}{0.017}$$

$$\boxed{\mu = 1.03 \text{ kg}}$$

④ $X \sim N(m, 560^2)$ $P(X > 8000) = 0.122$



$$z_1 = \frac{8000 - m}{560}$$



$$\Phi(z_1) = 0.878$$

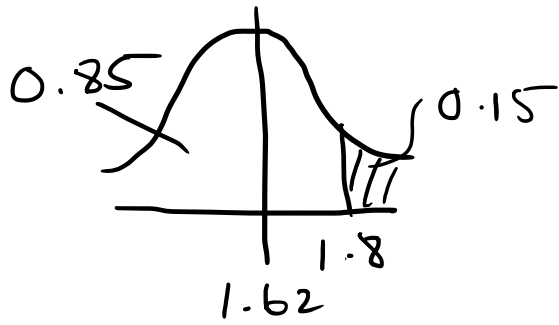
$$\therefore z_1 = 1.165$$

$$\therefore 1.165 = \frac{8000 - m}{560}$$

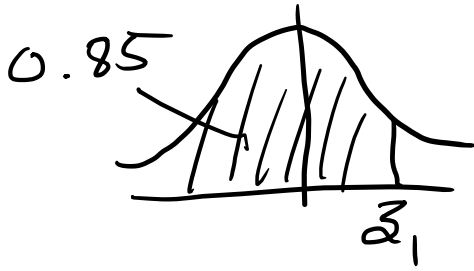
$$\therefore m = 7347.6$$

$$\therefore \boxed{m = 7350 \text{ sales}}$$

⑤ $X \sim N(1.62, \sigma^2)$ $P(X > 1.8) = 0.15$



$$\therefore z_1 = \frac{1.8 - 1.62}{\sigma}$$



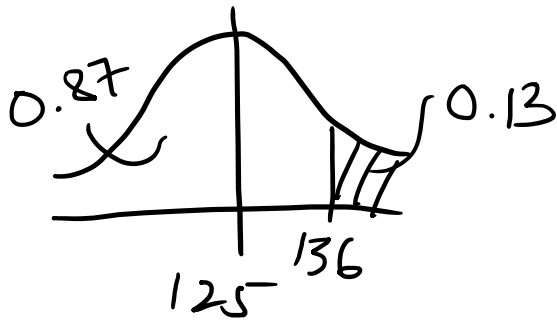
$$\therefore \Phi(z_1) = 0.85$$

$$z_1 = 1.037$$

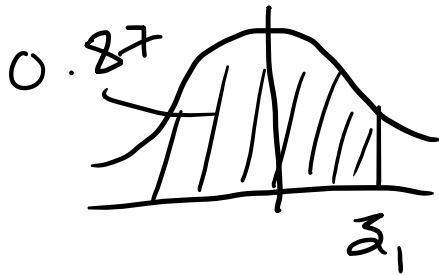
$$\therefore 1.037 = \frac{1.8 - 1.62}{\sigma}$$

$$\therefore \boxed{\sigma = 0.174 \text{ m}}$$

⑥ $X \sim N(125, \sigma^2)$ $P(X > 136) = 0.13$



$$Z_1 = \frac{136 - 125}{\sigma}$$



$$\therefore \Phi(Z_1) = 0.87$$

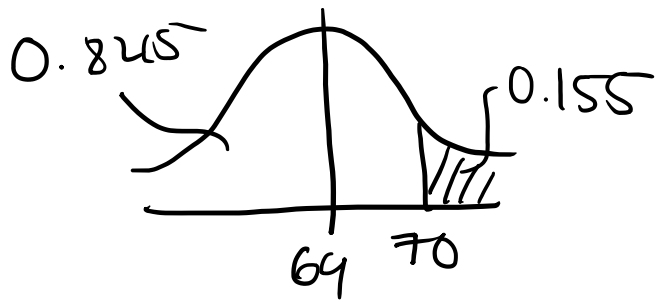
$$Z_1 = 1.127$$

(FROM TABLE)

$$\therefore 1.127 = \frac{136 - 125}{\sigma}$$

$$\therefore \boxed{\sigma = 9.76}$$

⑦ $X \sim N(69, \sigma^2)$ $P(X > 70) = 0.155$



$$\bar{z}_1 = \frac{70 - 69}{\sigma}$$



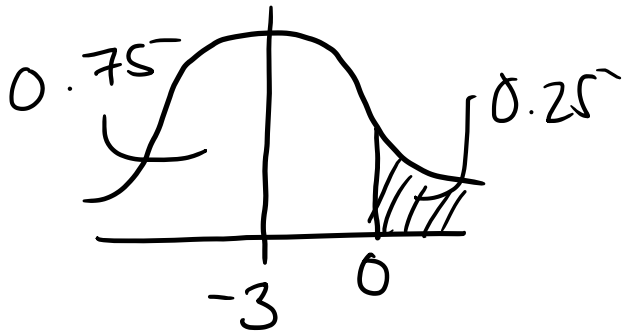
$$\Phi(\bar{z}_1) = 0.845$$

$$\bar{z}_1 = 1.015$$

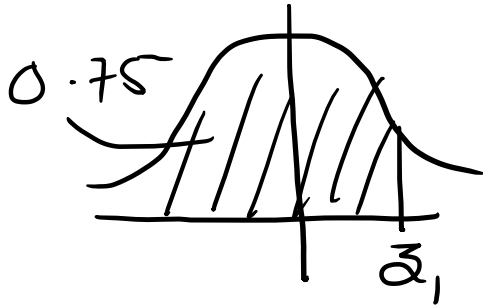
$$\therefore 1.015 = \frac{70 - 69}{\sigma}$$

$$\therefore \boxed{\sigma = 0.985 \text{ cm}}$$

⑧ $X \sim N(-3, \sigma^2)$ $P(X > 0) = 0.25$



$$z_1 = \frac{0 - (-3)}{\sigma}$$



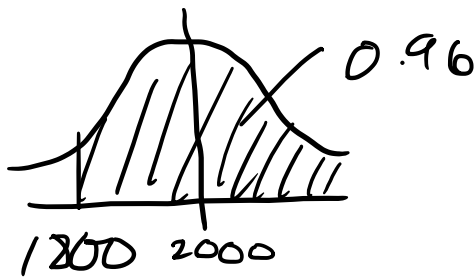
$$\Phi(z_1) = 0.75$$

$$z_1 = 0.674$$

$$\therefore 0.674 = \frac{3}{\sigma}$$

$$\therefore \boxed{\sigma = 4.45}$$

⑨ $X \sim N(2000, \sigma^2)$ $P(X > 1800) = 0.96$



$$z_1 = \frac{1800 - 2000}{\sigma}$$



$$\Phi(z) = 0.96$$

$$z = 1.751$$

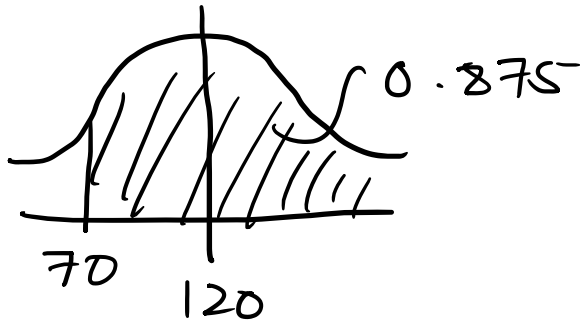
$$\therefore z_1 = -1.751$$

$$-1.751 = \frac{1800 - 2000}{\sigma}$$

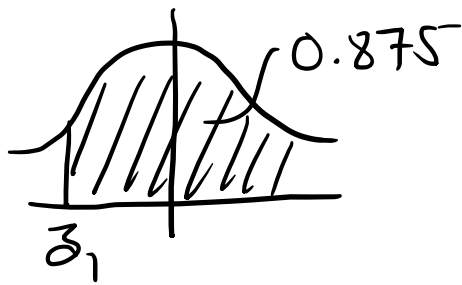
$$1.751 = \frac{200}{\sigma}$$

$$\therefore \boxed{\sigma = 114 \text{ hours}}$$

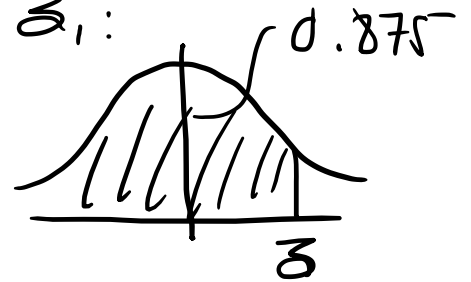
⑩ $X \sim N(120, 5)$ $P(X > 70) = 0.875$



$$z_1 = \frac{70 - 120}{5}$$



FIND z_1 :



$$\Phi(z) = 0.875$$

$$z = 1.151$$

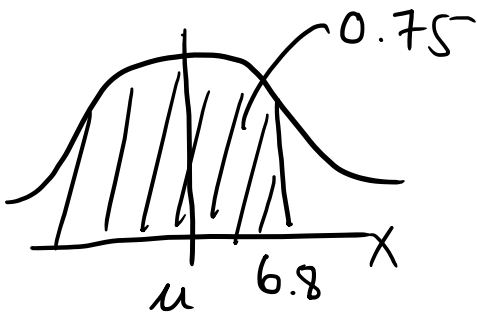
$$\therefore z_1 = -1.151$$

$$\therefore -1.151 = \frac{70 - 120}{5}$$

$$1.151 = \frac{50}{5}$$

$$\therefore \boxed{5 = 43.4 \text{ hrs}}$$

⑪ $X \sim N(\mu, \sigma^2)$ $\sigma = 0.25\mu$
 $P(X < 6.8) = 0.75$



$$\therefore Z = \frac{6.8 - \mu}{\sigma}$$

$$\Phi(Z) = 0.75$$

$$\therefore Z = 0.674$$

$$\therefore 0.674 = \frac{6.8 - \mu}{\sigma}$$

$$0.674 = \frac{6.8 - \mu}{0.25\mu}$$

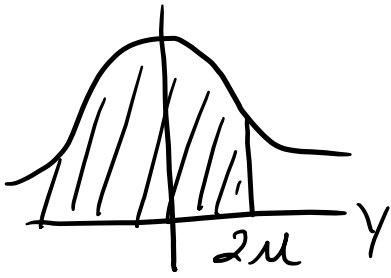
$$0.1685\mu = 6.8 - \mu$$

$$1.1685\mu = 6.8$$

$$\boxed{\mu = 5.82}$$

$$(12) Y \sim N(\mu, \sigma^2) \quad \sigma = \frac{2}{3}\mu$$

$$P(Y < 2\mu) = ?$$



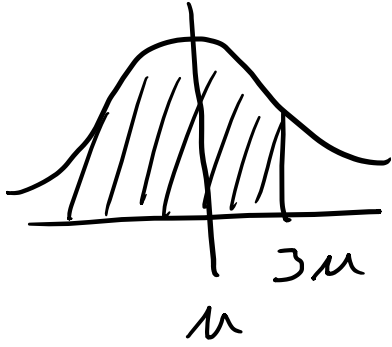
$$\begin{aligned} Z &= \frac{2\mu - \mu}{\sigma} \\ &= \frac{\mu}{\frac{2}{3}\mu} = \frac{3}{2} \end{aligned}$$

$$\therefore Z = 1.5$$

$$\begin{aligned} \therefore P(X < 2\mu) &= P(Z < 1.5) = \Phi(1.5) \\ &= 0.933 \end{aligned}$$

$$\boxed{P(X < 2\mu) = 0.933}$$

⑬ $X \sim N(\mu, \sigma^2)$ $3\sigma = 4\mu$



$$P(X < 3\mu) = ?$$

$$\therefore Z = \frac{3\mu - \mu}{\sigma} = \frac{2\mu}{\frac{4}{3}\mu}$$

$$Z = \frac{3}{2} = 1.5$$

$$\therefore P(X < 3\mu) = P(Z < 1.5) = \Phi(1.5) \\ = 0.933$$

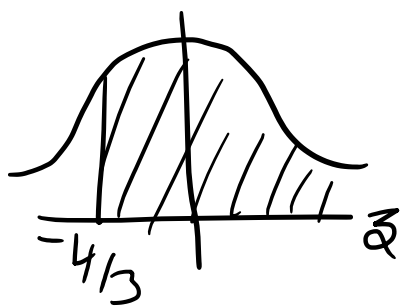
$$\therefore \boxed{P(X < 3\mu) = 0.933}$$

⑭ $Y \sim N(\mu, \sigma^2)$ $4\sigma = 3\mu$
 $P(Y > 0) = ?$

Corresponding
z-value : $z = \frac{0 - \mu}{\sigma} = \frac{-\mu}{\frac{3}{4}\mu}$

$$z = -\frac{4}{3}$$

$$\therefore P(Y > 0) = P(z > -\frac{4}{3})$$



$$= P(z < \frac{4}{3})$$

$$= \Phi(1.333) = 0.909$$

$$\therefore \boxed{P(Y > 0) = 0.909}$$

⑮ $X \sim N(\mu, \sigma^2)$ $\mu = 1.5\sigma$
 $P(X > 0) = ?$

$$Z = \frac{0 - \mu}{\sigma} = \frac{-\mu}{\frac{\mu}{1.5}} = \frac{-\mu}{\frac{2}{3}\mu}$$

$$\therefore Z = -\frac{3}{2}$$

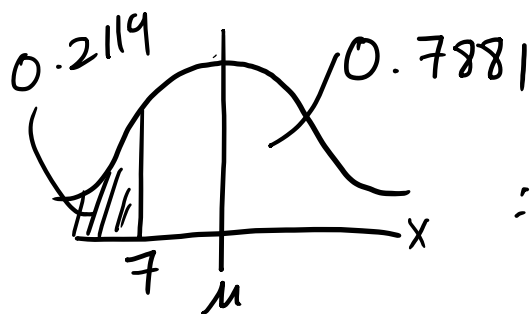
$$\begin{aligned}\therefore P(X > 0) &= P(Z > -\frac{3}{2}) = P(Z < \frac{3}{2}) \\ &= \Phi(\frac{3}{2})\end{aligned}$$

$$\therefore \boxed{P(X > 0) = 0.933}$$

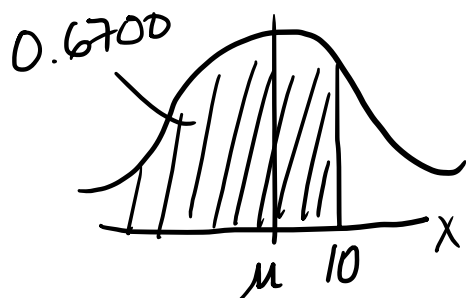
$$(16) X \sim N(\mu, \sigma^2)$$

$$P(X < 7) = 0.2119$$

$$P(X < 10) = 0.6700$$

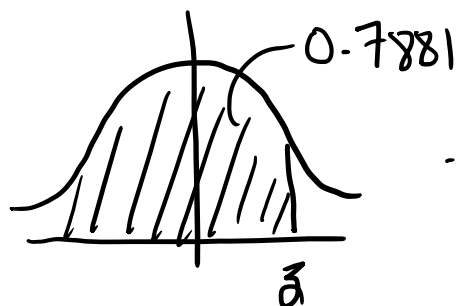


$$\therefore \bar{z}_1 = \frac{7 - \mu}{\sigma} \quad (1)$$



$$\bar{z}_2 = \frac{10 - \mu}{\sigma} \quad (2)$$

For $\bar{z}_1$:

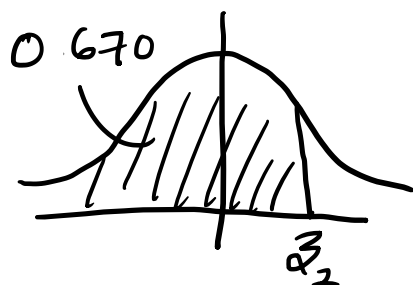


$$\therefore \Phi(\bar{z}) = 0.7881$$

$$\therefore \bar{z} = 0.80$$

$$\therefore \bar{z}_1 = -0.8$$

For $\bar{z}_2$:



$$\Phi(\bar{z}_2) = 0.6700$$

$$\therefore \bar{z}_2 = 0.44$$

$$\therefore (1) \quad -0.8 = \frac{7 - \mu}{\sigma}$$

$$-0.8\sigma = 7 - \mu$$

$$* \mu = 7 + 0.8\sigma$$

$$\textcircled{2} \quad 0.44 = \frac{10 - \mu}{\sigma}$$

$$0.44\sigma = 10 - \mu$$

$$* \mu = 10 - 0.44\sigma$$

$$\therefore \textcircled{1} \rightarrow \textcircled{2} \quad 7 + 0.8\sigma = 10 - 0.44\sigma$$

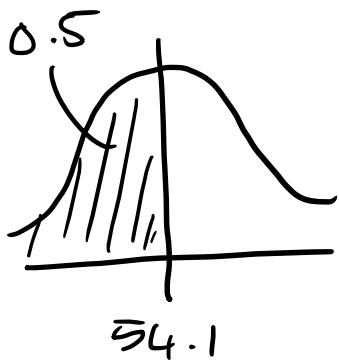
$$1.24\sigma = 3$$

$$\boxed{\sigma = 2.42}$$

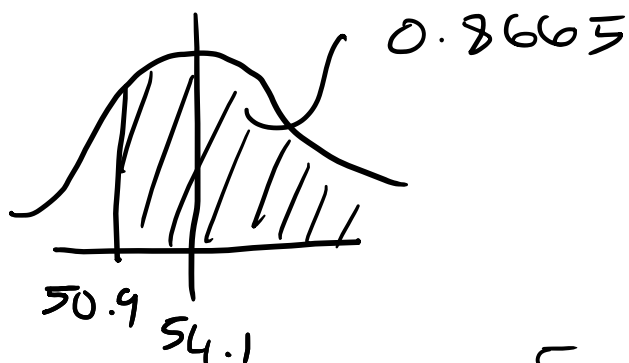
$$\therefore \textcircled{2} \quad \mu = 10 - 0.44(2.42)$$

$$\boxed{\mu = 8.94}$$

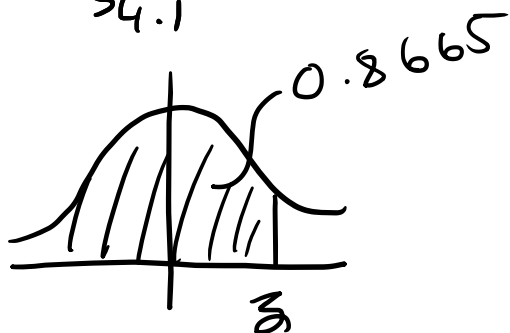
①⑦ $X \sim N(\mu, \sigma^2)$ $P(X < 54.1) = 0.5$
 $P(X > 50.9) = 0.8665$



$$\therefore \boxed{\mu = 54.1}$$



$$\therefore z_1 = \frac{50.9 - 54.1}{\sigma}$$



$$\Phi(z) = 0.8665$$

$$\therefore z = 1.1$$

$$\therefore z_1 = -1.1$$

$$\therefore -1.1 = \frac{50.9 - 54.1}{\sigma}$$

$$-1.1 \sigma = -3.2$$

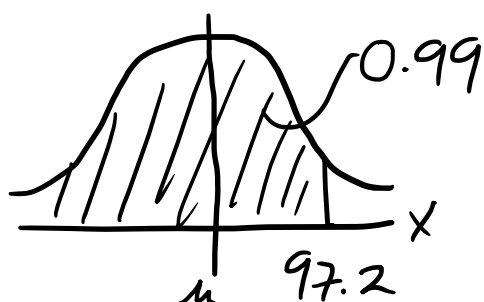
$$\therefore \boxed{\sigma = 2.88}$$

$$\boxed{\mu = 54.1}$$

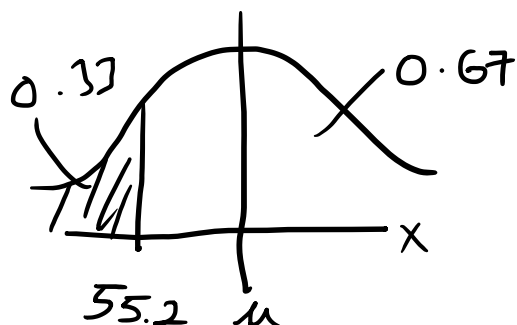
⑬ $X \sim N(\mu, \sigma^2)$

$$P(X < 97.2) = 0.99$$

$$P(X < 55.2) = 0.33$$



$$z_1 = \frac{97.2 - \mu}{\sigma} \quad (1)$$



$$z_2 = \frac{55.2 - \mu}{\sigma} \quad (2)$$

FOR z_1 : $\Phi(z_1) = 0.99$

$$\therefore z_1 = 2.326$$

FOR z_2 :



$$\Phi(z) = 0.67$$

$$\therefore z = 0.44$$

$$\therefore z_2 = -0.44$$

$$(1) : 2.326 = \frac{97.2 - \mu}{\sigma}$$

$$* \mu = 97.2 - 2.326\sigma$$

$$\textcircled{2} \quad -0.44 = \frac{55.2 - \mu}{\sigma}$$

$$* \mu = 55.2 + 0.44\sigma$$

$$\textcircled{1} \rightarrow \textcircled{2} \quad 97.2 - 2.326\sigma = 55.2 + 0.44\sigma$$

$$2.766\sigma = 42$$

$$\boxed{\sigma = 15.2}$$

$$\therefore \textcircled{2} \quad \mu = 55.2 + 0.44(15.2)$$

$$\boxed{\mu = 61.9}$$

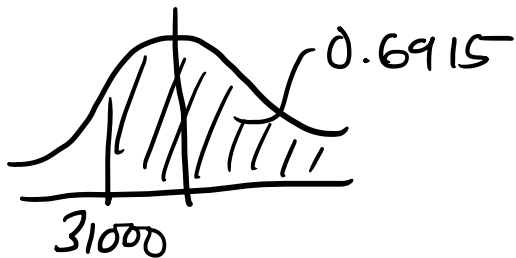
$$(19) \quad X \sim N(\mu, \sigma^2)$$

$$P(X > 36800) = 0.0082$$

$$P(X > 31000) = 0.6915$$



$$z_1 = \frac{36800 - \mu}{\sigma} \quad (1)$$



$$z_2 = \frac{31000 - \mu}{\sigma} \quad (2)$$

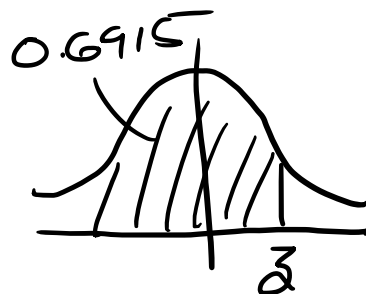
FIND z_1 :



$$\Phi(z_1) = 0.9918$$

$$z_1 = 2.4$$

FIND z_2 :



$$\Phi(z) = 0.6915$$

$$\therefore z = 0.5$$

$$\therefore z_1 = -0.5$$

(1)

$$2.4 = \frac{36800 - \mu}{\sigma}$$

$$* \mu = 36800 - 2.4\sigma$$

$$\textcircled{2} \quad -0.5 = \frac{31000 - \mu}{\sigma}$$

$$* \mu = 31000 + 0.5\sigma$$

$$\textcircled{1} \rightarrow \textcircled{2} \quad 36800 - 2.4\sigma = 31000 + 0.5\sigma$$

$$2.9\sigma = 5800$$

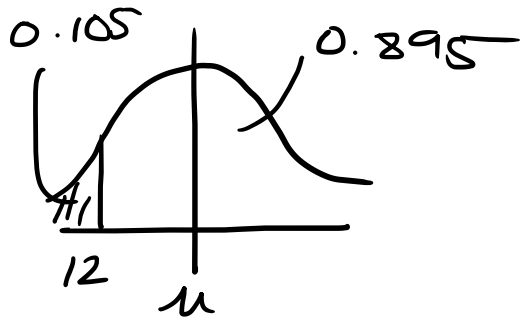
$$\boxed{\sigma = 2000}$$

$$\therefore \textcircled{2} \quad \mu = 31000 + 0.5(2000)$$

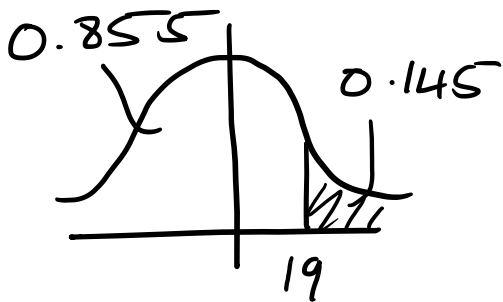
$$\boxed{\mu = 32000}$$

②① $X \sim N(\mu, \sigma^2)$ $P(X < 12) = \frac{42}{400} = 0.105$

$P(X > 19) = \frac{58}{400} = 0.145$

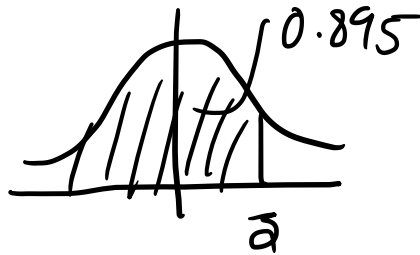


$z_1 = \frac{12 - \mu}{\sigma}$ ①



$z_2 = \frac{19 - \mu}{\sigma}$ ②

FIND z_1 :

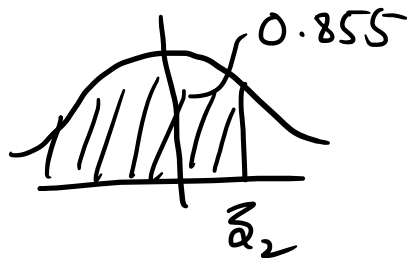


$\Phi(z) = 0.895$

$z = 1.253$

$\therefore z_1 = -1.253$

FIND z_2 :



$\Phi(z_2) = 0.855$

$z_2 = 1.058$

$\therefore$ ① : $-1.253 = \frac{12 - \mu}{\sigma}$

* $\mu = 12 + 1.253\sigma$

$$\textcircled{2} : 1.058 = \frac{19 - \mu}{\sigma}$$

$$* \mu = 19 - 1.058\sigma$$

$$\textcircled{1} \rightarrow \textcircled{2} \quad 12 + 1.253\sigma = 19 - 1.058\sigma$$

$$2.311\sigma = 7$$

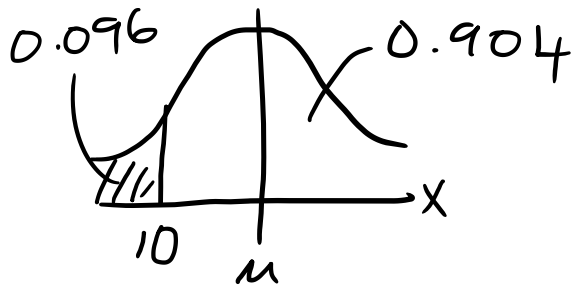
$$\boxed{\sigma = 3.03}$$

$$\therefore \textcircled{2} : \mu = 19 - 1.058(3.03)$$

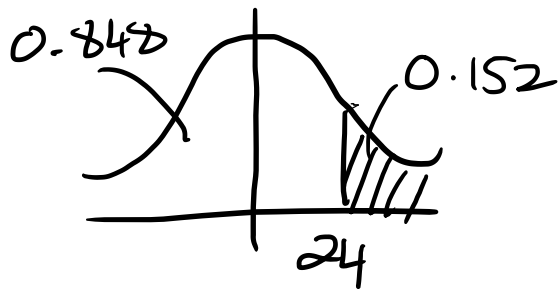
$$\boxed{\mu = 15.8}$$

$$\textcircled{21} \quad X \sim N(\mu, \sigma^2) \quad P(X < 10) = \frac{48}{500} = 0.096$$

$$P(X > 24) = \frac{76}{500} = 0.152$$

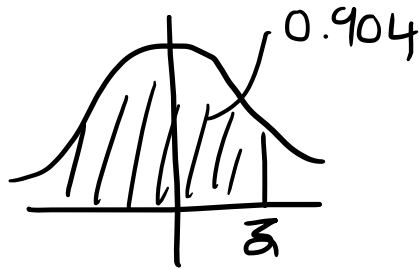


$$z_1 = \frac{10 - \mu}{\sigma} \quad \textcircled{1}$$



$$z_2 = \frac{24 - \mu}{\sigma} \quad \textcircled{2}$$

FIND z_1 :



$$\Phi(z_1) = 0.904$$

$$\therefore z_1 = 1.305$$

$$\therefore z_1 = -1.305$$

FIND z_2 :



$$\Phi(z_2) = 0.848$$

$$\therefore z_2 = 1.028$$

$$\therefore \textcircled{1} : \quad -1.305 = \frac{10 - \mu}{\sigma}$$

$$* \mu = 10 + 1.305\sigma$$

$$\textcircled{2} : 1.028 = \frac{24 - \mu}{\sigma}$$

$$* \mu = 24 - 1.028\sigma$$

$$\textcircled{1} \rightarrow \textcircled{2} \quad 10 + 1.305\sigma = 24 - 1.028\sigma$$

$$2.333\sigma = 14$$

$$\sigma = 6.00$$

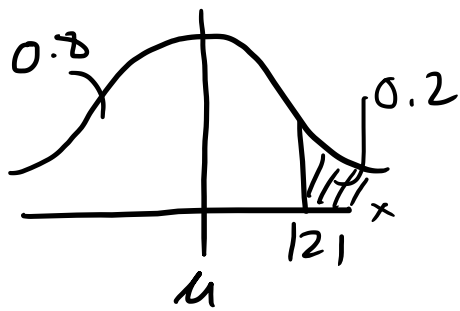
$$\therefore \boxed{\sigma = 6}$$

$$\textcircled{2} \quad \mu = 24 - 1.028(6.00)$$

$$\mu = 17.83$$

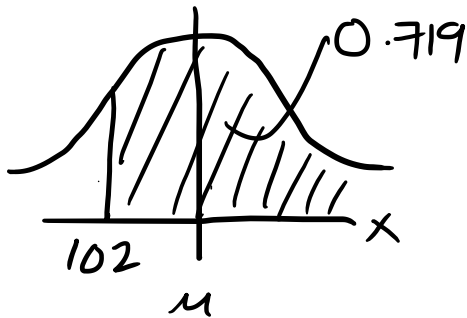
$$\therefore \boxed{\mu = 17.8}$$

②② $X \sim N(\mu, \sigma^2)$ $P(X > 121) = 0.2$



$$P(X > 102) = 0.719$$

$$Z_1 = \frac{121 - \mu}{\sigma} \quad (1)$$



$$Z_2 = \frac{102 - \mu}{\sigma} \quad (2)$$

FIND Z_1 : $\Phi(Z_1) = 0.8$

$$\therefore Z_1 = 0.842$$

FIND Z_2 : $\Phi(Z) = 0.719$

$$Z = 0.58$$

$$\therefore Z_2 = -0.58$$

$$\therefore \quad (1) \quad 0.842 = \frac{121 - \mu}{\sigma}$$

$$\therefore \mu = 121 - 0.842\sigma$$

$$\textcircled{2} \quad -0.58 = \frac{102 - \mu}{\sigma}$$

$$* \mu = 102 + 0.58\sigma$$

$$\therefore \textcircled{1} \rightarrow \textcircled{2} \quad 121 - 0.842\sigma = 102 + 0.58\sigma$$

$$1.422\sigma = 19$$

$$\therefore \boxed{\sigma = 13.4}$$

$$\textcircled{2} \quad \mu = 102 + 0.58(13.4)$$

$$\boxed{\mu = 110}$$